I3RC Monte Carlo Model: Phase 1

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This work presents algorithms of the Monte Carlo method, developed to calculate the fields of the mean and higher order moments of

(1) albedo R, transmittance T, and absorptance A; and

(2) upward $I^{\uparrow}$ and downward $I^{\downarrow}$ radiances in a given direction

at the corresponding boundaries of the horizontally (and vertically) inhomogeneous clouds occupying the layer $Hb \le z \le Ht$.

The model calculations are made under assumption that the optical depth $\tau(\vec{r})$, $\vec{r} = (x, y, z)$, the cloud top $Ht(\vec{r})$, and/or bottom $Hb(\vec{r})$ heights are piecewise constant functions in space:

- **1D model (X).** The cloud optical depth depends only on the x coordinate, and it is constant within each pixel: $\tau(\vec{r}) = \tau(x) = \tau_i$, $i = 1, \ldots nx$. The cloud top and bottom heights are constant: $Ht(\vec{r}) = Ht$, $Hb(\vec{r}) = Hb$. The cloud is infinite in the Y-direction.

- **2D model (XZ).** The optical depth depends on the x and z coordinates, and it is constant within each pixel: $\tau(\vec{r}) = \tau(x, z) = \tau_{i,k}$; the cloud top and bottom heights are defined as: $Ht(\vec{r}) = Ht_i$, $Hb(\vec{r}) = Hb_i$, $i = 1, \ldots nx$, $k = 1, \ldots nz$. Because it is possible that $\tau_{i,k} = 0$, we defined the cloud top height Ht as the highest level in the atmosphere, and cloud base height Hb as the lowest level where $\tau_{i,k} > 0$, $i = 1, \ldots nx$, $k = 1, \ldots nz$. The cloud is infinite in the Y-direction.

- **2D model (XY).** The optical depth and the cloud top height depend on the x and y coordinates, and they are constant within each pixel: $\tau(\vec{r}) = \tau(x, y) = \tau_{i,j}$, $Ht(\vec{r}) = Ht(x, y) = Ht_{i,j}$, $i = 1, \ldots nx$, $j = 1, \ldots ny$. The cloud base height is constant: $Hb = const$. The cloud is infinite in the X- and Y-directions.

The realizations corresponding to these models will be called Case 1, Case 2, and Case 3, respectively.

Calculations are made assuming:

- no atmospheric effects;

- Lambertianly reflecting underlying surface with albedo A_s;

- periodic (along the X (and Y) direction) boundary conditions;

- a unit parallel solar flux F incident on the cloud top in the direction $\vec{\omega}_{\oplus} = (\xi_{\oplus}, \varphi_{\oplus})$, where $\xi_{\oplus}$ and $\varphi_{\oplus} = 0^{\circ}$ are the solar zenith and azimuth angles.

The net horizontal flux H, nadir I_u and oblique I_o reflectivities, and zenith transmissivity (I_d) are calculated from the formulas

$$H = 1 - R - T \times (1 - A_s) - A,$$

$$I_{o(d)} = \left(\pi \times I^{\uparrow(\downarrow)}\right) \Big/ \left(F \times |c_{\oplus}|\right),$$

where $c_{\oplus} = \cos\xi_{\oplus}$.

Calculation techniques

The radiation calculations in clouds are made using numerical simulation technique and the method of local estimate [1].

Numerical simulation technique. The method essentially consists of the computer simulation of cloud field realization and the solution of radiative transfer equation in this realization. The method advantages are that

- it can be used to simulate cloud field realizations of any complexity, provided they can be constructed on computer; and

- it can estimate the radiative characteristics with prescribed accuracy without additional approximations and simplifications.

The method deficiency is that it consumes much computer time, especially when used to simulate too complex cloud realizations with irregular geometry or/and optical (microphysical) parameters.

Method of local estimate. The method of local estimate is based upon approach developed for estimating the linear functionals $J_h = (f, h)$ of solution of the integral equations of the second kind

$$f = Kf + \psi .$$

The general theory of Monte-Carlo dictates that a linear functional J_h is calculated by estimating the mathematical expectation of the random quantity η

$$J_h = M\eta = \sum_{n=0}^{N_0} Q_n h(\vec{x}_n), \qquad (1)$$

where

M is the mathematical expectation over the ensemble of particle trajectory realizations;

N_0 is random number of the last state in Markov chain, with the weights given as

$$Q_0 = 1,\; Q_n = w \times Q_{n-1},\;\; n \ge 1,$$

w is the single scattering albedo;

$\vec{x} = (\vec{r}, \vec{\omega}) \in X$, X is the phase space of coordinates and directions.

In [1] it is shown that the intensity of radiation passing through the plane $z = z'$ in direction $\vec{\omega}_* \neq \vec{\omega}_\oplus$ can be calculated from formula (1) by letting

$$h(\vec{r}_n, \vec{\omega}_n) = \begin{cases} \dfrac{g(\vec{\omega}_n, \vec{\omega}_*)}{|c_\bullet|} \exp\left(-\tau(\vec{r}_n, \vec{r}_{out}, \vec{\omega}_*)\right), \\ \quad if \;\; (z' - z_n) \times c_* > 0; \\ 0, \; (z' - z_n) \times c_* < 0 \end{cases} \qquad (2)$$

and, correspondingly, the fluxes can be calculated by

$$h(\vec{r}_n, \vec{\omega}_n) = \begin{cases} \exp\left(-\tau(\vec{r}_n, \vec{r}_{out}, \vec{\omega}_{n+1})\right), \\ if \;\; (z' - z_n) \times c_{n+1} > 0 \\ 0, \; (z' - z_n) \times c_{n+1} < 0 \end{cases} \qquad (3)$$

Here, the scattering phase function $g(\vec{\omega}, \vec{\omega}') = g(c)/2\pi$, $c = [\vec{\omega}' \cdot \vec{\omega}]$ and $\tau(\vec{r}, \vec{r}_{out}, \vec{\omega})$ is the optical path between point $\vec{r}$ and the point $\vec{r}_{out}$ located at the boundary of the cloud layer:

$$\vec{r}_{out} = \begin{cases} \vec{r} + \vec{\omega} \times (Ht - z)/c, \; c > 0, \\ \vec{r} + \vec{\omega} \times (Hb - z)/c, \; c < 0 \end{cases}$$

Upon reflection from the underlying surface, the particle weight is multiplied by A_s. Because the Lambertian surface reflects according to the c/π law, formula (2) now looks

$$h(\vec{r}_n, \vec{\omega}_n) = \begin{cases} \exp\left(-\tau(\vec{r}_n, \vec{r}_{out}, \vec{\omega}_*)\right), \; c_* > 0, \\ 0, \; c_* < 0 \end{cases} \qquad (2')$$

The trajectory modeling is made using standard algorithms [1]. For more efficient computation, in both techniques the particle free-path length is simulated using the method of maximal cross section. The idea of the method is to use, instead of the horizontally inhomogeneous medium, a surrogate scattering medium with a constant extinction coefficient σ_{max} such that

$$\sigma(\vec{r}) \le \sigma_{max}.$$

Using this, the direct simulation of the radiative transfer process leads us to a standard procedure of simulation of the free path length only between "physical" collisions [1].

The proposed statistical simulation techniques are advantageous in that (1) they can calculate the radiative characteristics simultaneously for the sets of single-scattering and surface albedos, and calculate the radiance for those same sets of parameters and, in addition, for a number of directions; and (2) they save current computation results, at the end of each batche, to a temporary file. Using this file, it is possible either to proceed with the computation until reaching a desired accuracy or finish computation that was occasionally interrupted. Such a computation scheme significantly improves the algorithm efficiency.

Accuracy of results

We used the direct simulation technique to calculate the radiative fluxes for all cloud realizations considered above. The radiance in a given direction was calculated by the method of local estimate.

The present calculations were made using Pentium (120 MHz) and Pentium II (450 MHz). Table 1 gives the number of photons, mean pixel level errors, and errors of mean for each of the three Cases.

Table 1. The number of photons, mean pixel level errors (MPLE), and errors of mean (ME) for Case 1, Case 2, Case 3.

Case	**Experiments**	R, T, A		$I^{\uparrow(\downarrow)}$	
		No. of photons $\times 10^6$	MPLE / ME, %	**No. of photons $\times 10^6$**	MPLE / ME, %
1	1-4	50	0.15 / 0.02	50	0.37 / 0.07
2	1-5	300	0.30 / 0.01	**200**	**0.7** / 0.03
	6-8	200	0.35 / 0.01	**100**	**2.5** /0.12
3	1-4	500	**1.1** / 0.01	500	**1.8** /0.02

Case 1. Each simulation was performed using 50×10^6 photons so that the top of each column received, on average, $1{,}56\times10^6$ photons. The cloud field realization considered here is quite simple, so this number of photons is sufficient to calculate

the flux and radiance fields with acceptable mean pixel-level errors: 0.15% for R, T, A and 0.37% for $I^{\uparrow(\downarrow)}$.

In addition, the mean values of R, T, A, and H, as well the corresponding radiative fields, are also calculated by method of local estimate. Comparison of results, calculated by different methods using same number of photons, shows good agreement (Table 2). In this simple realization the methods differ little in efficiency.

Case 2. With the average number of photons per column (~ 460,000 in 1 through 5 experiments and ~ 310,000 in 6 through 8experiments) used to calculate the flux fields, the mean pixel-level error did not exceed ~0.3%. Unfortunately, the photons used to calculate radiance fields was too few to get acceptable accuracy, especially in the 6 through 8 experiments, performed with the scattering phase function for C_1 cloud. In these experiments, ~156,000 photons per column on average have been used, which gave the maximum mean pixel-level error of ~2.5%. Therefore, we found it unreasonable to submit our nadir reflectivity and zenith transmissivity fields for intercomparison.

Table 2. Means / second-order central moments (SOCM) of R, T, A and H, calculated by the direct simulation technique and by the method of local estimate for Case 1. Given in parentheses are errors of the mean for R, T, and A (%). ($A_s = 0$, $w = 0.99$, $\xi_{\oplus} = 60^{\circ}$, $50{\times}10^6$ photons).

		The method of local estimate	The direct simulation technique
R	Mean / SOCM	0.4765 (0.01) / 0.0291	0.4764 (0.01) / 0.0291
T	Mean / SOCM	0.3248 (0.02) / 0.0047	0.3249 (0.02)/0.0047
A	Mean / SOCM	0.1986 (0.01) / 0.0306	0.1986 (0.01) / 0.0306
H	Mean / SOCM	0 / 0.1222	0 / 0.1224

Case 3. While making computations, we had opportunity to see how radiation fields depend on the number of photons used to calculate them. The optical depth τ of the cloud realization in Case 3 has quite strong gradients both along OX- and OY-axes (Fig. 1a). Figure 1b presents the difference between fields of albedo R calculated using $300{\times}10^6$ (R_{300}) and $500{\times}10^6$ (R_{500}) photons ($\xi_{\oplus} = 0$, $w = 1$). The comparison indicates that, for the fragments of the cloud realization in which optical depth varies fairly smoothly, the difference $\Delta R = R_{300} - R_{500}$ is rather small: $|\Delta R| \le 0.004$. The number of simulated photons becomes more important in the

realization fragments where τ has large gradients: $|\Delta R|$ increases to $|\Delta R| \approx 0.02$.

With computer resources available to us presently, we ran 500×10^6 photons so that the top of each column received, on average, ~30,000 photons. Mean pixel-level error was 1.1% for R, T, A fields and 1.8% for $I^{\uparrow(\downarrow)}$ fields. Obviously, for better quality of radiation-field calculations, it is necessary to either use more photons to achieve required accuracy, or elaborate on the existing algorithms, allowing them to calculate cloud fields with complex optical-geometrical structure more efficiently.

Conclusions

Phase 1 of I3RC was a test of monochromatic radiative transfer through clouds in the empty atmosphere. It should help to identify reasons for possible discrepancies between different calculation methods. The next step for Phase 2 would be to include experiments that incorporate additional atmospheric components such as aerosols and atmospheric gases. However, before investigating the combined 3D effects of clouds and effects associated with gaseous and aerosol absorption (and aerosol scattering), it seems advisable to intercompare 1D plane-parallel models. This would help to estimate the discrepancies arising due to use of the different treatments of gaseous absorption.

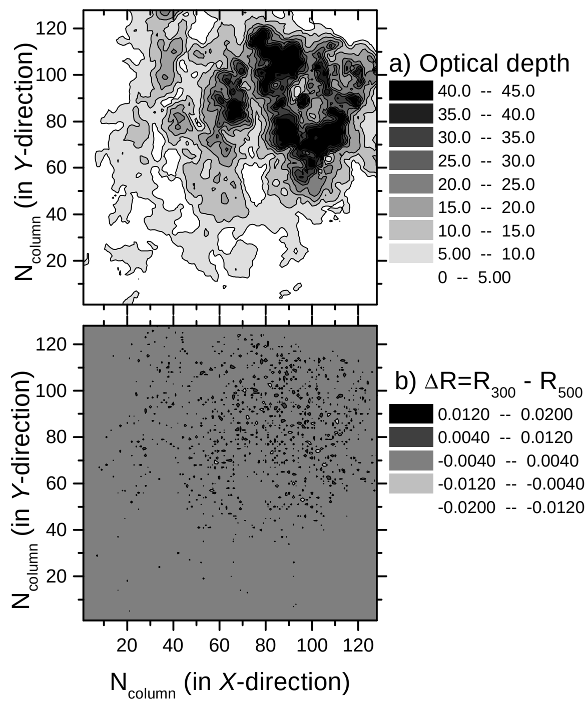

Fig. 1. Case 3: dependence of the reflected radiance fields on the number of photons used for simulation ($\xi_{\oplus} = 0,\ w = 1$).

References

1. Marchuk G.I., G.A. Mikhailov, M.A. Nazaraliev, R.A.Darbinyan, B.A. Kargin, and B.S. Elepov, Monte-Carlo Method in Atmospheric Optics, 283 pp., Nauka, Novosibirsk, Russia, 1976.