Background to GEODYN Modeling of Diurnal/Semidiurnal Tidal Polar Motion

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As of today, the input of polar motion parameters to GEODYN has been revised. The polar motion parameters now follow quite closely the paper by Chao et al. (1996), which was the original source of the model used in the IERS-2010 Conventions. This memo gives some of the algebraic background for how these parameters are entered into GEODYN and how they are internally employed by GEODYN.

Unfortunately, the IERS uses conventions for Earth orientation parameters that are objectionable in several ways and do not follow standard conventions from the ocean or earth-tide communities. This often makes for confusion when deriving EOP parameters from ocean-tide models, or when converting the IERS's coefficients for our own use. The coefficients in this memo can be used as a guide for future conversions, since the major constituents listed below should be (approximately) consistent with the model used in the IERS-2010 conventions.

GEODYN employs a prograde/retrograde formulation for polar motion. (It also employs a prograde/retrograde formulation for the spherical harmonic coefficients—in OTIDE cards—of the tides themselves.) This formulation is another source of potential confusion, since simple (x, y) pole coordinates might be simpler and in fact are used internally by GEODYN. However, a strong reason to maintain the prograde/retrograde formulation arises when polar motion is being estimated; it is often the case that the space-geodetic data are more sensitive to either prograde or retrograde motion. This has long been the case when estimating low-degree spherical harmonic components of the tides from SLR data, since it is only the prograde terms that cause the long-period orbit perturbations that SLR is most sensitive to. Chao et al. (1996) also used a prograde/retrograde formulation for polar motion.

The current IERS-2010 model corresponds to Model C in the Chao-96 paper. The Chao-96 tables list amplitudes and phases of prograde (A_p, α_p) and retrograde (A_r, α_r) polar motion for 8 major tidal constituents. What GEODYN requires on input are the in-phase and quadrature components

$A^{\pm}, B^{\pm}$ given quite simply by

$$\begin{array}{lll} A^+ = A_p \cos\alpha_p & B^+ = A_p \sin\alpha_p & \text{prograde} \\ A^- = A_r \cos\alpha_r & B^- = A_r \sin\alpha_r & \text{retrograde} \end{array}$$

For example, for the M_2 tidal constituent Chao-96 lists:

$$(A_p, \alpha_p) = (75\ \mu\text{as},\ 116^\circ) \qquad (A_r, \alpha_r) = (263\ \mu\text{as},\ 271^\circ),$$

so the GEODYN input is:

$$\begin{array}{rrcrcrl} \text{prograde:} & A^+ & = & -32.9\,\mu\text{as} & = & -1.59\times10^{-10} & \text{radians} \\ & B^+ & = & 67.4 & = & 3.27\times10^{-10} & \\ \text{retrograde:} & A^- & = & 4.6 & = & 0.22\times10^{-10} & \\ & B^- & = & -263.0 & = & -12.75\times10^{-10} & \end{array}$$

For all the constituents tabulated by Chao-96, the transformation from their tabulated parameters to what GEODYN requires proceeds exactly as the above expressions for M_2. The whole set is given here in Table 1 in GEODYN input format.

In addition to these 8 constituents, accurate tidal prediction requires accounting for a number of additional minor constituents that should not be ignored. We have computed coefficients for these minor constituents by using standard methods of tidal inference, including accounting for the core resonance in the diurnal band (Ray, 2017). If the need arises, it is easy to extend the inference calculations to include even more, smaller constituents.

Note that GEODYN internally applies 18.6-year nodal modulations to all lunar tides. Thus, our GEODYN setup should never include tidal lines that correspond to these nodal sidelines. In contrast, the IERS explicitly lists these sidelines. (It is easy to recognize a nodal sideline: its 5th Doodson digit differs from the main line by either ±1 or ±2. There are also some 8.8-year perigean sidelines, but these occur only for the relatively minor constituents L_2 and M_1.)

We now turn to how these polar-motion parameters are used internally by GEODYN. Most users need not be concerned with this, but it is useful to put together some documentation on this while the approach and the calculations are still fresh in memory.

Table 1: GEODYN polar motion input for Chao-96 Model C*

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OLOAD  1     2    135655       2.9446024E-11  1.2754474E-10          0.0     0.0  Q1+
OLOAD  1     2    145555       2.3545878E-10  6.4691769E-10          0.0     0.0  O1+
OLOAD  1     2    163555       1.2325645E-10  2.4190441E-10          0.0     0.0  P1+
OLOAD  1     2    165555       3.7637238E-10  7.3867238E-10          0.0     0.0  K1+
OLOAD  1     2    245655      -5.8278557E-11  5.8278557E-11          0.0     0.0  N2+
OLOAD  1     2    255555      -1.5939625E-10  3.2681074E-10          0.0     0.0  M2+
OLOAD  1     2    273555       9.8074789E-12  1.4025348E-10          0.0     0.0  S2+
OLOAD  1     2    275555      -3.5473780E-12  3.3751047E-11          0.0     0.0  K2+
OLOAD  1     2    845655      -3.8075244E-12 -2.1813293E-10          0.0     0.0  N2-
OLOAD  1     2    855555       2.2252865E-11 -1.2748658E-09          0.0     0.0  M2-
OLOAD  1     2    873555       2.9816417E-10 -5.6076525E-10          0.0     0.0  S2-
OLOAD  1     2    875555       8.4897152E-11 -1.4129259E-10          0.0     0.0  K2-
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[*] Our “standard” setup for this model also includes many additional constituents inferred from these 8 major constituents. Altogether, those coefficients then correspond approximately to the same model as the IERS-2010 recommends (“approximately” because the IERS went through several additional transformations before they arrived at their final coefficients, whereas we start with Chao-96 directly; also our methods of inferring minor constituents differ slightly).

Note that Chao-96 used a right-handed coordinate system such that the p_y polar motion is in the direction of the 90°E meridian. However, most space-geodetic users and the IERS, as well as GEODYN, take p_y in the direction of 90°W. We will thus follow that convention in what follows. In complex form, the polar motion for any given tidal constituent is

$$p = p_x - ip_y \tag{1}$$

and can be written in prograde/retrograde components as

$$p = A_p \exp\{i\alpha_p + i\Theta(t)\} + A_r \exp\{i\alpha_r - i\Theta(t)\}. \tag{2}$$

The function $\Theta(t)$ is the tidal argument in Doodson's standard convention based on a linear combination of astronomical mean longitudes, and including various offsets of ±90° in the diurnal band or ±180° in the semidiurnal—see Doodson & Warburg (1941) or Table 4.1 of Pugh & Woodworth (2014)[1]. Expanding out (2) gives

$$\begin{aligned}
p_x &= A_p \cos\alpha_p \cos\Theta - A_p \sin\alpha_p \sin\Theta + A_r \cos\alpha_r \cos\Theta + A_r \sin\alpha_r \sin\Theta \\
&= \cos\Theta(A_p \cos\alpha_p + A_r \cos\alpha_r) + \sin\Theta(-A_p \sin\alpha_p + A_r \sin\alpha_r) \\
&= \cos\Theta(A^+ + A^-) + \sin\Theta(-B^+ + B^-) \qquad (3) \\
-p_y &= A_p \sin\alpha_p \cos\Theta + A_p \cos\alpha_p \sin\Theta - A_r \cos\alpha_r \sin\Theta + A_r \sin\alpha_r \cos\Theta \\
&= \cos\Theta(A_p \sin\alpha_p + A_r \sin\alpha_r) + \sin\Theta(A_p \cos\alpha_p - A_r \cos\alpha_r) \\
&= \cos\Theta(B^+ + B^-) + \sin\Theta(A^+ - A^-). \qquad (4)
\end{aligned}$$

As an aside, we note that the components multiplying $\cos\Theta$ are simply the in-phase components of either p_x or p_y, and those multiplying $\sin\Theta$ are the quadrature components:

$$\begin{aligned}
p_x &= A_x \cos\Theta + B_x \sin\Theta \\
p_y &= A_y \cos\Theta + B_y \sin\Theta,
\end{aligned}$$

so that

$$\begin{aligned}
A_x &= A^+ + A^- \qquad & B_x &= -B^+ + B^- \\
A_y &= -B^+ - B^- \qquad & B_y &= -A^+ + A^-.
\end{aligned}$$

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[1] But watch for a typographical error in Pugh-Woodworth's argument of K_1.

This shows how to convert between x/y and prograde/retrograde coefficients of polar motion.

Equations (3–4) are very nearly in the form that GEODYN uses to evaluate p_x, p_y, but not quite. Whenever a retrograde component is being used, GEODYN flips the sign of the tidal argument, replacing Θ with $\Theta' = -\Theta$. Thus,

$$\begin{aligned} p_x &= A^+ \cos\Theta + A^- \cos\Theta' - B^+ \sin\Theta - B^- \sin\Theta' & (5) \\ p_y &= -B^+ \cos\Theta - B^- \cos\Theta' - A^+ \sin\Theta - A^- \sin\Theta' & (6) \end{aligned}$$

is the formulation used inside GEODYN. Equations (5–6) have the nice property that the algebra is the same for both prograde and retrograde terms. That is, we have

$$\begin{aligned} p_x &= A\cos(\mathsf{arg}) - B\sin(\mathsf{arg}) \\ p_y &= -B\cos(\mathsf{arg}) - A\sin(\mathsf{arg}) \end{aligned}$$

for both prograde and retrograde components, so long as the argument arg is set equal to Θ for prograde polar motion and Θ' for retrograde.

References

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